The capacitated mobile facility location problem (CMFLP) is defined on a network where clients and facilities are initially located at vertices on the network. Associated with each client is a demand and each facility has a specified capacity available to service demand. A destination vertex must be determined for each facility and each client should be assigned to one of the facilities so that the total demand of the clients assigned to a facility respects the capacity. The objective is to minimize the total weighted distance traveled by the facilities and the clients.

Formally, the CMFLP is set on a graph $G = (V, E)$ where V denotes the set of vertices and E denotes the set of edges. A non-negative distance d_{ij} is defined for each edge $(i, j) \in E$. We interchangeably use cost and distance henceforth to indicate d_{ij}. The initial locations of the clients are represented by the subset $C \subseteq V$. Each client $i \in C$ has demand q_i and a positive weight u_i. There are different types of facilities with differing capacities. Each facility is of a type from the set T and the subset $F = \bigcup_{t \in T} F_t \subseteq V$ of vertices denotes the initial locations of the facilities (so F_t denotes the set of initial locations of facilities of type t). Each facility $j \in F_t$ has capacity Q_t and a positive weight w_j for relocation. All facilities are equipped with the same capabilities and therefore a client can get service from any one of them as long as the capacity restrictions are satisfied. In a feasible solution to the CMFLP, each facility $j \in F$ moves to a destination vertex $v(j) \in V$ and each client $i \in C$ moves to a destination vertex $v(i) \in V$ with the condition that $v(i) = v(j)$ for some j. A facility cannot share a destination vertex with another facility and a client can only be served by a single facility, i.e. demand cannot be split. Total demand assigned to a type t facility cannot exceed Q_t, for all $t \in T$. Clients or facilities may stay put (i.e., have their destination equal to their origin). Clients and facilities are also permitted to start at the same vertex. The objective is to minimize the total weighted distance traveled by the facilities and the clients, that is, $\sum_{j \in F} w_j d_{j,v(j)} + \sum_{i \in C} u_i d_{i,v(i)}$.

By including the capacity restrictions, the CMFLP extends the mobile facility location problem (MFLP) introduced previously by Demaine et al. [2009] to a practical setting. The CMFLP finds applications in logistics planning of community outreach programs delivered via mobile facilities. Examples of community outreach programs that utilize mobile facilities include library outreach programs in rural areas, mobile daycare delivered to farm children, and mobile schools that provide basic education to street children, as well as temporary schools servicing refugee camps. The deployment of mobile healthcare facilities (e.g. cancer screening units, blood banks, eye clinics, vaccination booths in case of a disease outbreak) that serve beneficiaries residing in either urban districts or rural regions is another important application area of the CMFLP. In these applications, districts (population centers) that have patients residing in them are represented by client vertices in the CMFLP. Mobile medical facilities currently located at some of the districts are represented by facility vertices. The demand of a district shows the number of patients and their demands (i.e., visits to the medical facility) in the district and the capacity of a facility is the total number of patient visits it can handle within a time frame. Weights may be assigned to facilities and client locations according to priority, patient criticality, number of patient visits, etc. The objective of the problem is to move the mobile facilities so that every patient is served and the total weighted distance traveled by the facilities and the patients is minimized. After demand is served in an area or demand patterns have significantly changed, facilities may be relocated to a new area. The facility destinations in the previous network will be the originating facility vertices in the current network. Then, the problem can be solved with new clients and their respective demands.

The importance of mobile facilities is noted both in the medical and the operations research communities. Geoffroy et al. [2014] discuss the benefits of mobile healthcare facilities as a complementary service to fixed clinics by expanding access to healthcare for hard-to-reach areas. However, studies addressing location decisions for healthcare facilities focus mainly on fixed clinics and hospitals, and typically aim to maximize coverage of demand locations. For example, Verter and

Lapierre [2002] model the preventive healthcare facility location problem as an extension of the Maximal Coverage Location Problem. Ha et al. [2013] discuss applications of the multi-vehicle covering tour problem related to deployment of mobile healthcare teams and mobile library teams and the distribution of relief items after a disaster. The problem involves choosing the stops of the vehicles from a set of potential locations so that every person can reach one of these stops within an acceptable time limit. The CMFLP differs from these studies significantly as it addresses capacity limitations of the facilities while minimizing the total distances traveled by both the facilities and the clients.

Our Contributions: In this paper, we develop exact and heuristic algorithms to solve the CMFLP. We first compare two (linear) integer programming (IP) formulations for the CMFLP. The first formulation, which we call the *layered graph* formulation, extends the one given in Halper et al. [2015] for the MFLP to account for the capacity constraints. The second formulation is a *set partitioning* formulation where each variable corresponds to a type of facility to be moved to a vertex in order to serve a feasible set of clients (i.e. the total demand of the clients cannot exceed the capacity). We prove that the LP relaxation of the set partitioning formulation provides a lower bound to the CMFLP that is greater than or equal to the LP relaxation bound from the layered graph formulation and can be strictly better. Next, we provide a branch-and-price algorithm for the set partitioning formulation where a column generation procedure is used on the set partitioning formulation to obtain lower bounds. Furthermore, we present two heuristic approaches for the CMFLP. The first is an LP rounding heuristic that is also used to obtain good quality upper bounds within the branch-and-price algorithm. The second is a local search heuristic called 1-OptSwapBI that is adapted from one of the local search heuristics described in Halper et al. [2015].

To show the efficacy of the branch-and-price algorithm and the underlying column generation procedure, we conducted computational tests on instances adapted from Halper et al. [2015] (where each vertex hosts a client). We found out the ratio of the number of clients to the number of facilities plays an important role on the performance of both the branch-and-price algorithm and the heuristics. We solved the layered graph formulation using CPLEX as a benchmark. We observe that in general the problem is harder to solve when the average number of clients per facility is relatively small (i.e., the ratio of $|C|$ to $|F|$ is small). However, in these instances the branch-and-price algorithm outperforms the CPLEX benchmark. Furthermore, the local search heuristic complements the branch-and-price algorithm by obtaining good solutions quickly when the average number of clients per facility is larger.

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