An Exact Approach for the Vehicle Routing Problem with Location Congestion

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Abstract

The Vehicle Routing Problem with Location Congestion (VRPLC) integrates Vehicle Routing Problem (VRP) and the location congestion constraints (e.g., due to docking capacity). The capacity of any location that needs to be served by multiple vehicles is assumed to be limited, so that only a limited number of vehicles can visit a location in any particular time period. The goal for VRPLC is to minimize the weighted summation of vehicle travel time between visited locations and vehicle waiting time induced by the visited locations congestion. We formulate the problem into a Mixed Integer Programming (MIP) model and propose an exact approach using Benders decomposition to solve it.

VRPLC is a common problem in shopping mall products delivery and inventory replenishment. Vehicles from multiple logistic service providers (LSPs) deliver goods and products to the shopping mall everyday. The loading/unloading activities are performed in the docking area of each shopping mall (i.e., visited location). The docking slots in shopping malls are limited and becoming the bottleneck for the whole delivery process. If a vehicle cannot find a docking slot when it arrives at a shopping mall, we assume that it must wait for an available slot there. This leads to some extra waiting time for the vehicle, especially in the case of non-organized routes and schedules selected by non-cooperative LSPs. The goal of VRPLC is to reduce the travel time and extra waiting time by optimizing the routes and schedules for the vehicles of all LSPs. VRPLC has received little attention in the literature and very few researchers have explored the problem. The closest study on our topic is by Lam and Van Hentenryck (2016). They solved a PDP (pickup and delivery problem) with location resource constraints, with an objective of minimizing the travel distance. The difference with the VRPLC we study is that we are minimizing both the travel time and the extra waiting time.

Our approach is capable of managing the vehicle fleets of all the LSPs and docking areas of all the shopping malls, thereby enabling LSPs to perform more deliveries with lower costs.

We first formulate the problem to a Mixed Integer Programming (MIP) model. The inputs to the model include:

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- LSPs vehicle fleet size, capacity and available time period
- Shopping malls dock capacity and available time period
- Delivery request (pickup/delivery location, service time)
- Travel time between different locations (malls)

In the interest of space, the pseudo mathematical model is as shown below:

$$\min \ (travel\ time\ +\ waiting\ time) \tag{1}$$

Subject to:

$$vehicle\ flow\ constraints \tag{2}$$

$$vehicle\ capacity\ constraints \tag{3}$$

$$docking\ capacity\ constraints \tag{4}$$

We propose an exact approach using Benders decomposition to solve the above MIP model. The approach decomposes the initial problem to a master problem, which is a typical VRP, and a subproblem, which is a non-linear problem that is difficult to get the traditional Benders cut. To solve the subproblem efficiently, we formulate it to a Constraint Programming (CP) and relaxed Linear Programming (LP). For a given solution from the master problem, we can solve the CP based subproblem and generate feasibility cut (no-good cut) if none feasible solution can be found; if any feasible solution can be identified, we can solve the LP based subproblem and generate optimality Benders cut.

In the numerical experiments, we solve a set of random instances. In each instance, the number of shopping malls (visited locations) is 5, and the number of orders is between 30 to 150. If we only optimize the routes as a typical VRP, the vehicles will create congestion at the shopping malls during peak hours. In contrast, with considerations of docking capacity, if we optimize both the routes and schedules using the exact approach to the VRPLC, we can save around 5% travel time and waiting time for vehicles in the proposed instances. Our proposed approach demonstrates good performance and outperforms the state-of-art MIP solvers (e.g. CPLEX) that are applied directly on the VRPLC.

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References

Edward Lam and Pascal Van Hentenryck. A branch-and-price-and-check model for the vehicle routing problem with location congestion. *Constraints*, 21(3): 394–412, 2016.