Vehicle routing with space- and time-dependent stochastic travel times

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Abstract

Vehicle travel times in real-world road networks are stochastically dependent in space and time. That is, the travel time on one link in one period will be correlated to travel times on the same link in nearby time periods, as well as travel times on neighboring links in the same time period. Space- and time-dependent stochastic travel times have not been considered in the vehicle routing (VRP) literature, as far as we know. Dependent travel times lead to a very high dimensional dependent random vector, and increases the complexity of the corresponding VRPs substantially.

VRPs are usually solved by improvement heuristics, which improve candidate solutions continuously based on some initial candidate solutions. However, to the best of our knowledge, there is presently no way to evaluate candidate solutions to a VRP with space- and time-dependent stochastic travel speeds. The focus of this paper is on the objective function evaluation for given solutions to this type of VRP. We shall discuss how such VRPs should be modeled in time and space, how the random variables can be represented, how scenarios can be generated to be used in a stochastic program, and how different objective functions can affect the stability of the scenario generation method. In particular we shall focus on the difficulties arising from the dimension of the random vector. The larges case we consider has, for example, over 310 million correlations. Obviously that cannot be handled by any direct methodology.

Numerical experiments are given and tested. For a case with 142 nodes, 418 road links and 60 periods, we end up with 25,080 dependent random variables. To achieve an accuracy of 1%, we need only ten scenarios. It indicates that it is not always true that the number of scenarios will *likely be quite large* for stochastic VRPs as expressed by Gendreau et al. (2016, Transportation Science 50 (4), 1163-1173). The reason is that in a VRP we end up with routes which have a very nice structure relative to how we generate scenarios. So the number of random variables can be huge, but typically not the number of scenarios.