A bi-criteria approach for the DARP with stochastic travel times

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Time plays a critical role in goods and persons transportation due to limited time availability at the delivery points, to time-based quality of service for the deliveries and to synchronization at the intermediate facilities, among others. Thus, computing solutions on such logistic systems can become significantly harder. The situation is even more complicated when traffic conditions – for instance daily congestions due to commuting – are taken into account by means of time-dependent or stochastic travel times. Namely, a solution computed a static context may break time constraints when subjected to variations on travel time. This specially happens in urban context where variability is usually higher.

The Dial-a-Ride Problem (DARP) is an extension of the Capacitated Vehicle Routing Problem (CVRP) in which several transportation requests have to be handled by a fleet of vehicles. The problem is defined on a complete connected digraph $G = (V, A)$ corresponding to the transportation network. Each request consists of a group of q_i clients to be transported by one vehicle from the pickup node $i^+ \in V$ to the delivery node $i^- \in V$. Given an arc $(i, j) \in A$, t_{ij} and c_{ij} are respectively the transportation time and the transportation cost to travel directly from i to j. For each node $i \in V$, a time window $[e_i; l_i]$ sets respectively the earliest and the latest starting time of service, and the service duration is d_i. The fleet of vehicle is located at a depot, also subjected to time windows. The problem consists in finding a set of routes to handle all the transportation requests while satisfying all the constraints, including the vehicle capacity, the time windows on each node and a maximal riding duration for each request. The objective is to minimize the total transportation cost. This problem is central in the transportation of persons, for instance transporting by a dedicated fleet elderly or disabled persons to/from the hospital. It is NP-hard and many exact and heuristics methods have been proposed, see [3].

We consider here the Stochastic DARP (SDARP), in which the travel times are stochastic in order to model the variability in urban traffic conditions. For any arc $(i, j) \in A$, the transportation time t_{ij} is replaced by a positive random variable T_{ij}. We use a chance-constraint reformulation to model the SDARP and we propose a way to approximate the solution robustness, *i.e.* the probability a solution can handle variability in the time constraints. Then we propose a bi-criteria optimization: the first criterion is the total transportation cost and the second criterion is the solution robustness.

Given a SDARP solution s, the robustness estimator ρ_s is defined as $\rho_s = \prod_{r \in s} \rho_r$, where ρ_r is the robustness estimator of route $r \in s$. In turn, ρ_r is defined as the probability of all time constraints in the route to hold. The linear-time algorithm from [4] to check if a route r satisfies all the DARP constraints is modified in order to check if a route r satisfies all the SDARP constraints with a probability at least ρ. Then, using dichotomy on ρ, the largest $\rho_r \in [0; 1]$ such that r satisfies all the SDARP constraints can be computed. A bi-objective heuristic based on the NSGA-II algorithm [2] is used to compute a front of non-dominated solutions. NSGAII relies on a population of individuals. Here, each individual corresponds to a full sequence of requests, decoded into a DARP solution using a constructive heuristic. The crossover and the mutation operators are defined on the sequences of requests in order to build new sequences. The population evolution follows the scheme defined by [2].

The experiments have been done on computer with an Intel i7-3770 CPU at 3.40GHz. The set of 20 instances from [1] is used. A Normal distribution $\mathcal{N}(\mu_{ij_i}, \sigma_{ij}^2)$ is used for each arc $(i, j) \in A$ to build the SDARP instance out of the DARP instance. Here, $\mu_{ij} = t_{ij}$ and $\sigma_{ij} = t_{ij}/10$. From the Pareto front, the solution with the smallest transportation cost (leftmost solution) and the solution with the highest robustness (rightmost solution) are reported. in Table 1. For each extreme solution (leftmost / rightmost), the total transportation cost $c(s)$, its relative gap to the best know DARP value, the value of the robustness estimator ρ and the evaluation $\rho^*$of the robustness by simulation are reported. The time in seconds spent by NSGA-II is reported in last column.

		leftmost solution				rightmost solution				
instance	$c(BKS)$	$c(s)$	*gap*	ρ	ρ^*	$c(s)$	*gap*	ρ	ρ^*	time
pr01	190.02	190.02	0.00%	74.8%	87.3%	197.42	3.89%	100.0%	100.0%	14.95
pr02	301.34	311.51	3.37%	97.7%	99.9%	317.95	5.51%	100.0%	100.0%	74.75
pr03	532.00	556.60	4.62%	50.7%	71.0%	622.37	16.99%	100.0%	100.0%	137.54
pr04	570.25	620.31	8.78%	69.8%	62.2%	641.66	12.52%	100.0%	100.0%	440.73
pr05	626.93	675.98	7.82%	30.7%	16.1%	691.59	10.31%	100.0%	100.0%	721.79
pr06	785.26	833.78	6.18%	7.0%	0.3%	857.31	9.17%	100.0%	100.0%	1310.82
pr07	291.71	297.36	1.94%	72.2%	75.5%	304.29	4.31%	100.0%	100.0%	28.11
pr08	487.84	505.43	3.61%	33.5%	37.3%	546.28	11.98%	100.0%	100.0%	160.26
pr09	658.31	717.53	9.00%	45.0%	61.9%	758.78	15.26%	100.0%	100.0%	672.75
pr10	851.82	918.80	7.86%	12.7%	4.4%	1005.33	18.02%	100.0%	100.0%	1275.53
pr11	164.46	168.51	2.46%	72.1%	68.9%	171.44	4.24%	100.0%	100.0%	16.74
pr12	295.66	304.18	2.88%	19.0%	0.5%	312.65	5.75%	100.0%	100.0%	81.93
pr13	484.83	520.95	7.45%	93.0%	96.8%	531.94	9.72%	100.0%	100.0%	221.26
pr14	529.33	574.28	8.49%	45.9%	34.7%	584.81	10.48%	100.0%	100.0%	609.96
pr15	577.29	589.36	2.09%	96.7%	48.4%	601.13	4.13%	100.0%	100.0%	1191.81
pr16	730.67	781.06	6.90%	30.1%	16.4%	803.26	9.94%	100.0%	100.0%	1932.74
pr17	248.21	262.41	5.72%	21.8%	1.4%	273.73	10.28%	100.0%	100.0%	34.68
pr18	458.73	482.44	5.17%	95.9%	100.0%	514.56	12.17%	100.0%	100.0%	258.34
pr19	593.49	642.90	8.32%	32.9%	51.9%	677.33	14.13%	100.0%	100.0%	743.31
pr20	785.68	860.79	9.56%	22.9%	2.5%	935.25	19.04%	100.0%	100.0%	1880.71
avg.	508.19	541.06	5.65%	51.5%	46.7%	567.45	10.39%	100.0%	100.0%	590.44

Table 1: NSGA-II results on instances from [1]

NSGA-II is able to find solutions (leftmost solution) that are not too far from the best known DARP solutions. The average gap is 5.65% but the average associated robustness is quite low (46.7%). The robustness can be much improved, as seen on the rightmost solution. It is close to 100% while the relative gap from the best solution does not deteriorate much.

Our approach is generic enough to be able to handle a large number of distribution laws, including but not limited to the normal and the shifted gamma distributions. Besides, with no modification, it could handle correlated random variables to address predictable congested urban areas. Our approach can also be adapted to other urban transportation problems, either for persons or for goods.

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[2] Deb K, Pratap A., Agarwal S. and Meyarivan T., A fast and elitist multi-objective genetic algorithm: NSGA-II. IEEE Transactions on Evolutionary Computation. Vol. 6(2). pp. 182–97. 2002.

[3] Doerner K.F., Salazar-Gonzalez J.-J., Pickup and Delivery Problems for People Transportation, in Toth P., Vigo D. editors, Vehicle Routing: Problems, Methods, and Applications, 2nd edition, SIAM, Monographs on Discrete Mathematics and Applications, pp. 193-212, 2014.

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