Solving the Consistent Vehicle Routing Problem via Column Generation

Kai Wang, Lu Zhen, Jun Xia*, Shuaian Wang

(*Corresponding email: lgtxiaj@sjtu.edu.cn)

Consistent vehicle routing problem (ConVRP) aims at creating a set of route schedules for homogeneous drivers to fulfill the delivery demands on multiple days. The route schedules have to follow the driver consistency (DC) such that each customer is served by a limited number of familiar drivers, and the time consistency (TC) such that each customer is visited at roughly the same time on each day when there is a demand. The objective of ConVRP is to minimize the route cost while taking into account both DC and TC to improve customer satisfaction.

Optimizing ConVRP is computationally challenging, as it is at least as hard as solving a periodic vehicle routing problem, which is known to be strongly NP-hard (Gaudioso and Paletta, 1992). There is recent but limited body of work, such as those in (Groër et al., 2009, Kovacs et al., 2014, 2015, Smilowitz et al., 2013), have addressed consistency issues in the vehicle routing. Most proposed models are based on arc-flow formulations whose linear programming (LP) relaxations can hardly provide tight bounds. As a result, existing solution methods proposed for solving the ConVRP primarily focus on finding heuristic solutions while the solution optimality gaps are seldom reported.

Column generation (CG) has successful applications in solving vehicle routing problems especially for those of large sizes. However, when applying CG to tackle the ConVRP, we find that solving a set-partitioning model that is directly derived from its arc-flow model can be ineffective. This mainly stems from the fact that the consistency related constraints in its relaxation formulation are often inactive, causing the pricing problems hardly generate high-quality columns to constitute even feasible schedules for drivers. This work strives to provide the ConVRP with a new set-partitioning model, based on which we can adopt the CG to design an effective and efficient method for solving this problem.

Our contributions are summarized as follows.

1. We develop for the ConVRP a new set-partitioning model which provides a tighter bound than those provided by existing arc-flow models. The proposed model also extends the original problem to simultaneously fulfill collection and distribution demands.

2. We propose a CG approach that effectively solves the LP relaxation problem of the proposed model. In order to make CG work more efficiently, we reformulate the master problem by including fewer variables and constraints. A bidirectional label-setting algorithm embedded with newly developed dominance rules is proposed to solving the pricing problems. Our CG approach is further integrated to find optimal and near-optimal route schedules for the ConVRP.

3. We examine our CG based method on a subset of small and medium benchmark instances provided by Groër et al. (2009). The experimental results show that all small instances (with 10 and 12 customers) can be solved optimally within several minutes. For the medium instances (with 50 customers) being tested with one hour run time limit, we can obtain feasible solutions which are competitive with the reported solutions in the literature.

References

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