Multi-Commodity Two-Echelon Vehicle Routing Problem with Time Windows

F. Dashti Saridarq, N. Dellaert, T. Van Woensel and T.G. Crainic

The two-echelon vehicle routing problem (2E-VRP) is a two-echelon distribution system where goods are transferred to customers by using intermediate facilities (called satellites). There are three types of facilities that are located in a hierarchical structure: (i) Depots act as the sources of goods and are located at the first level; (ii) Satellites are used to consolidate goods and are located at the second level; (iii) Final customers are located at the last level and are destinations of the goods. The first echelon of the 2E-VRP consists of vehicle routes where each route starts (and ends) at a depot and delivers a subset of goods to a subset of satellites. The second echelon consists of vehicle routes where each route starts and ends at the same satellite and delivers goods to their associated customers.

Most of the existing studies in the field of 2E-VRP have focused on the basic variant of the problem without origin-destination structure for demands, i.e, all demands are considered fully substitutable. In practice, this assumption is not realistic. Clearly, solutions derived from existing models to the 2E-VRP could be infeasible when implemented in real life. For example in parcel delivery application, each parcel should be picked up from a specific origin and be delivered to a specific customer destination and thus the parcels are non-substitutable. Therefore, the decision makers usually deal with origin-destination demands where each demand starts from a specific depot and is supposed to be delivered to a specific customer. A demand is a unique load (a specific parcel or package). We take non-substitutable demands into account by introducing commodities: a *commodity* consists of the destination customer, origin depot, a specific volume, and a time window which the delivery should take place within.

The described problem is referred as the *multi-commodity two-echelon vehicle routing problem with time windows* (MC-2E-VRPTW). Although some papers introduced the concept of non-substitutable demand [1], this is the first paper that considers non-substitutable origin-destination demands for the 2E-VRP. Each commodity (a demand request of a specific customer which should be transfered from a specific depot) should be shipped employing an urban vehicle (first echelon vehicle) and a city freighter (second echelon vehicle) which are connected through a satellite. A demand request consists of the size of the demand and also a hard time windows (the delivery of the commodity is not allowed either before the start of the window or after the end of this window).

To sum up, the *contributions* of this paper are as follows:

1. The literature on 2E-VRP has only focused on the single commodity variant of the problem which does not represent the non-substitutable demands arises in city logistics. We introduce the MC-2E-VRPTW which considers origin-destination structure for demands by employing multiple commodities. We investigate the problem from an exact optimization point of view and propose mathematical formulations and a branch-and-price algorithm. Specifically:

   (a) A combined arc-and-path based formulation is designed where arc-flow variables are used for the first echelon and path variables are used for the second echelon (denoted as the multi-commodity two-echelon combined arc-and-path based formulation, or *MC2E-A-P*). Particularly, it employs arc-flow variables for the first echelon routing problem and path variables for the second echelon routing problem. Note that, in practice, the number of satellites and depots is usually less than the number of customers. Therefore, the number of constraints to represent urban vehicle (first echelon vehicles) tours is not very huge. The full formulation will be discussed in the presentation and the full paper.

   (b) A branch-and-price algorithm is proposed which works on this formulation. A column generation method and specific branching strategies are introduced for the proposed algorithm. The proposed algorithm is based on a column generation method (see [3] for an overview). Two heuristics and one exact algorithm are used to solve a pricing problem to generate negative reduced cost city freighter tours.

2. The numerical evaluation through a large set of instances demonstrates the power of the newly developed model and the respective solution method. We study how large instances can be solved using the proposed branch-and-price algorithm for the arc-path-based formulation. Detailed results and insights based on an extensive instance set (inspired based on [2]) will be provided on the conference and in the full paper.

References

[1] T. G. Crainic, N. Ricciardi, and G. Storchi. Models for evaluating and planning city logistics systems. *Transportation Sci.*, 43(4):432–454, 2009.

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[3] M. E. Lübbecke and J. Desrosiers. Selected topics in column generation. *Oper. Res.*, 53(6):1007–1023, 2005.