The Vehicle Routing Problem with Distribution Uncertainty in Deadlines

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Abstract

In this paper, we investigate a vehicle routing problem (VRP) with probability constraints which require customers to be served before their uncertain deadlines to a specified level of service. It is unrealistic to expect the perfect knowledge on the probability distribution of deadlines is always available. To this end, we propose a distributionally robust optimisation framework to study worst bounds of the problem, which exploits the moment information of the historical observations. This framework includes two steps. We first use Conditional Value-at-Risk (CVaR) as a risk approximation to the probability of missing customer deadlines. The resulted nonlinear model is then transformed to a mixed integer program using the dual form of the CVaR approximation. We also study the convergence of the approximation with the increased number of observations. The extensive numerical experiments based on a number of variants of Solomon's benchmarks show that the routing policies developed by the proposed solution framework are robust and able to achieve the required levels of

service, regardless of deadline distributions. We also develop an iterative algorithm to reduce the risk approximation errors.

Our study investigates the stochastic VRPD from a different perspective. First, unlike Adulyasak and Jaillet (2014) and Jaillet et al. (2014), we consider uncertain deadlines and propose a distributionally robust optimization framework to address them, where the deadline at each demand point is defined as the latest time by which a vehicle must arrive. In some real world problems, the vehicles' velocities or the travelling times are comparatively more predictable than the latest available time of customers or the deadlines for services. These deadlines in practice are usually triggered by some random events and cannot be precisely predicted before deploying the vehicles.

Another difference from the previous research on the VRPD is the probability constraints considered in our problem, which specify an upper bound to the percentage of missed deadlines for all demand points. To address the computational difficulties caused by these probability constraints, we approximate them by exploiting the concept of Conditional Value-at-Risk (CVaR), which offers a nice convex structure and computational efficiency to the problem (see Hong et al. (2011), Anderson et al. (2016)). If the CVaR approximation is deemed as a tool for measuring the risk of missing deadlines, we have essentially incorporated the risk management ideas into the SVRP, which is an intuitive way to model the lateness of vehicles' arrivals. To the best of our knowledge, this is the first work which incorporates risk measurements into the SVRP. More importantly, our proposed approach could readily extend to including the third and higher order moments, while most of the previous works such as Adulyasak and Jaillet (2014) or Jaillet et al. (2014) are restricted to the first two moments.

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